Dynamic and Future-Oriented Dimensioning of Stock

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Meeting the objective of providing a high service level while at the same time keeping inventory levels to a minimum represents a great challenge to companies, particularly those operating in volatile markets. This article proposes a dynamic approach towards a future-oriented method of inventory control. The dynamic flow of safety stock can be determined on the basis of forecast data and the quality thereof together with supplier reliability. This will enable companies to manage their stock in a manner which is both future-oriented and transparent while being geared to serving their requirements.

JEL Codes: M11, L60 and C44

1. Introduction

The increasingly dynamic nature of today's markets means that short delivery times and a high degree of delivery reliability can often only be ensured by maintaining appropriate inventory levels (Ruiz-Torres 2010). Whether the focus is on stocks of raw materials, semi-manufactures or finished products is of secondary importance. The fundamental objective is in all cases to maintain a certain service level with respect to the subsequent buyer, whether this is a producer or consumer. The central challenge facing the task of inventory management in a company is hence to ensure an appropriate service level while at the same time taking into account economic aspects (i.e. keeping capital commitment costs to a minimum) (Inderfurth and Jensen 2008). The logistic objectives of higher service level coupled with minimised inventories are in constant competition with each other, a situation which has been referred to as the dilemma of inventory management (Nyhuis and Wiendahl 2009; Hopp and Spearman 2009). In order to adopt the right position within this state of conflict, inventory parameters need to be determined, particularly when it comes to safety stock. Several methods have been devised to determine these, all of which differ considerably in terms of their basic assumptions and complexity (cf. for instance Tempelmeier and Günther 2005; Gudehus 2006; Lutz et al. 2003; Eppen and Martin 1988).

Simulation studies undertaken by the Institute of Production Systems and Logistics (IFA) at the Leibniz University of Hannover have shown that these methods lead to widely differing levels of logistic efficiency, depending on the specific exogenous factors affecting the company. Logistic efficiency is measured in terms of the service level attained and the required mean inventory level. Exogenous factors, such as the dynamic forces operating in

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the sales market or supplier reliability have such a significant effect on the parameters of each respective method that it has not been possible to identify the optimum procedure by which to calculate safety stock.

Rather, whether or not a procedure can be classified as advantageous with respect to others in the situation at hand depends on temporary exogenous factors (Schmidt et al. 2012). The cause of this essentially stems from the fact that the majority of methods draw on statistical parameters based on historical data from previous periods. For instance, the dispersion or average value of demand is frequently calculated for an investigation interval. Although it is the case that several methods aim to counter market dynamics by means of dynamic safety stock calculations, they merely use static calculations embedded in a rolling plan. They are therefore regarded as quasi-dynamic.

This article presents the results of the project funded by the German Research Foundation (DFG) entitled ‘Developing a Method for Dynamic, Future-Oriented Dimensioning of Stock’. This dynamic method is based both on ongoing, continually changeable forecasts and on information regarding supplier reliability. Thus, the presented research findings are different from previous studies which focused on static or quasi-dynamic methods. The results obtained enable a future-oriented, dynamic management of safety stock and reorder levels at continuous demands. This will enable companies to position themselves within the dilemma of inventory management, even in dynamic markets. This dynamic method is based both on ongoing, continually changeable forecasts and on information regarding supplier reliability. The following section presents a literature review of existing approaches for calculating safety stocks. Afterwards the service-level operating curve and the derivation of the dynamic formula are introduced. The analysis section provides a possible experiment design of a simulation study and expected advantages of the introduced method. This is followed by a summary and conclusion.

2. Literature Review

The stock in a warehousing stage is controlled by means of so-called inventory policies. These determine the quantity of an article to be procured or produced and the points in time at which these quantities are to be available. The literature (Toomey 2002) distinguishes between four main types of inventory policy, which differ in terms of the following degrees of freedom: reorder point and reorder quantity. Reordering can either be done at fixed intervals or flexibly, in accordance with a certain rule. Similarly, the reorder quantity may be fixed or variable. Error! Reference source not found. shows the resulting characteristic inventory flows encountered in the (s,q)-inventory policy.
Based on mathematical considerations and simulation studies, it arises that the (s,q) inventory policy enables the required delivery capability at optimum cost (Jodlbauer 2008). Hence the (s,q)-policy forms the basis of the development times. To apply the (s,q)-policy, the parameters reorder quantity (q) and reorder level (s) need to be determined. Suitable deterministic methods of defining the most economical reorder level already exist (for example: Andler, Harris, Wagner-Within etc.). An overview is available in Nyhuis (1991).

The reorder level, on the other hand, is obtained by adding a safety stock to the consumption level in the replenishment time. Besides the lot stock level, safety stock is another central component that firstly guarantees security of supply in a warehousing stage and secondly has a decisive effect on the logistic costs above the level of the stock.

The following presents the existing approaches and methods for determining safety stocks, subdivided into the categories of 'heuristic approaches', 'deterministic methods', and 'mathematical-stochastic approaches'.

2.1 Heuristic Approaches

Heuristic approaches are generally simple in their application and are hence widespread in practice. The results of their application are neither analytically nor scientifically grounded but return a value based on experience (Alicke 2005). The simplest method is for the inventory manager to determine the required safety stock based on experience and estimates. This value is only to an extent based on analytical values or forecasts and is rather more determined by the gut feeling of the inventory manager. This approach can frequently lead to problems in the event of failure to adjust the safety stock level in the event that new products are introduced or old ones are discontinued or if there are changes in market levels. This can result in excessive warehousing costs and, in extreme cases, delivery bottlenecks coupled with extensive disturbances in the supply chain.

Such heuristic approaches do not permit systematic positioning in the context of the dilemma of inventory management as described above. The quality of the safety stock levels determined depends to a great extent on the experience of the relevant employees.

2.2 Deterministic Methods

In business economics, an attempt is always made to render the conflict in the target inventory management system accessible to evaluation and, in turn, to an unambiguous solution, by considering monetary factors. For this purpose, the two cost blocks of shortfall costs and inventory costs are placed in comparison with each other (Reichmann 2006). Inventory and shortfall costs – depending on the service level – run in opposing directions. The difficulties in quantifying shortfall costs make it virtually impossible to determine a position purely by taking into account cost considerations resulting from the conflict of objectives between low stocks and high parts availability. Due to the opposition between shortfall costs and inventory costs with respect to key figures representing parts availability (e.g. service level), business-economic concepts suggest an unambiguously defined optimum cost. However, this does not address the question posed from the point of view of inventory management relating to the size of the safety stock.
2.3 Mathematical-Stochastic Approaches

The field of mathematical-stochastic approaches offers numerous calculation rules for determining a safety stock, which take into account all similar input parameters and therefore display a similar basic orientation. Two methods are presented as examples, which differ considerably in their complexity. The basic formula should be regarded as a rudimentary method while the approach devised by Gudehus incorporates significantly more parameters.

The calculation rule frequently referred to in the literature as the standard formula for calculating safety stocks assumes a deterministic replenishment time and a stochastic consumption within this time (formula 1). The safety stock is determined by obtaining the product of the dispersion of consumption in the replenishment time and a safety factor (cf. for instance Schönsleben 2007, Alicke 2005, Eppen and Martin 1998):

\[ SSL = \sigma_d \cdot SF(SL) \]  

(1)

Where

- \( SSL \) = Safety stock level [units]
- \( \sigma_d \) = Standard deviation of demand during replenishment time [units]
- \( SF \) = Safety factor depending on service level [-]
- \( SL \) = Service level [%]

The premise of the normal distribution of demand should be viewed critically. Moreover, only the dispersion of demand is taken into account, while disturbing factors such as deviations in delivery dates or quantities are not taken into consideration in the calculation.

The approach taken by Gudehus (2006) results from a combination of stochastic demand dispersion during a deterministic replenishment time and the contrasting case of a deterministic demand within a stochastic replenishment time. As a rule, the time between two orders is greater than the replenishment time. In the period following the receipt of a delivery up until the time at which a new order is triggered, the warehouse has been filled and achieves a service level of 100%. It is only during a replenishment cycle that there is a risk of a stock shortage. The ratio of replenishment quantity to demand during the replenishment time is multiplied by the factor \((1 - \text{service level})\). This intermediate result is in turn subtracted from 1, and represents a low service level factor, from which the safety factor can be derived:

\[ SSL = SF \left(1 - \frac{(1 - SL) \cdot QRP}{TRP \cdot RD_m}\right) \cdot \sqrt{TRP \cdot \sigma_{RD}^2 + \sigma_{TRP}^2 \cdot RD^2} \]  

(2)

\forall QRP > TRP \cdot RD_m

Where

- \( QRP \) = Replenishment quantity [units]
- \( TRP \) = Replenishment time [SCD]
- \( RD_m = \text{Mean demand rate} \) [units/SCD]
- \( \sigma_{RD} = \text{Standard deviation of demand rate} \) [units]
- \( \sigma_{TRP} = \text{Standard deviation of replenishment time} \) [SCD]
- \( SF = \text{Safety factor depending on service level} \) [-]
- \( SL = \text{Service level} \) [%]
- \( SCD = \text{Shop calendar day} \)
Even when applying this expanded formula, the quality of the results with respect to the deviations in demand and replenishment times depends on the similarity with which they correlate with reality. The formula described here, in turn, requires normally distributed parameters. This condition for stock issue distributions is only fulfilled to a certain extent in practice (Axsäter 2006; Alicke 2005). With other forms of distribution, the quality of the results is questionable. Moreover, this formula makes use of historical values, as with the aforementioned standard formula.

The existing common methods for determining safety stocks are generally of a static nature and are based on simplifications (for instance with regard to the distribution form of important parameters). On the one hand, they only lead to approximated results, but on the other, they produce a simple, generally robust solution that is borne out in practice, when employed in a stable corporate environment. However, they do not measure up to the increasing momentum of important influencing factors such as trends or variation in demand or supplier reliability (Hartmann 2011). In order to fill this research gap in the literature this paper introduces a new approach to calculate safety stock taking discrete forecast data and supplier reliability into account. This ensures an accurate calculation of stock according to variation in demand and to variation of replenishment time.

3. Methodology and Model

Appropriate logistic positioning within the conflict of objectives between a high level of service on the one hand and low inventory levels on the other can be attained by using the service-level operating curve developed by Lutz (2002), which describes the service level with respect to the mean inventory level; an example is given in Error! Reference source not found.

![Figure 2: Characteristic operating points of service-level operating curves](image-url)

The shape of the service-level operating curve is essentially determined by two characteristic operating points: the lot stock level $BL_0$ and the stock level limit $BL_1$. The lot stock level is obtained by halving the incoming stock size, which describes the average inventory level excluding safety stock. Under ideal conditions, all demands can be served from the lot stock level, since there is sufficient stock available in the warehouse at all
times. Under realistic conditions, however, there are various disturbances and deviations from plan that impact on the stock, resulting in a reduction in service level. In order to nevertheless ensure 100% delivery capability, it is necessary to maintain a defined safety stock (SSL_{100%}). The stock level limit BL_{1} is obtained by adding this safety stock to the lot stock level. This represents the stock level from which the warehouse’s delivery capability is guaranteed at all times. From this point, the service level is at its maximum level (100%) and cannot be subjected to any further increase.

The deviations from plan that must be taken into consideration in practice result both from deviations in the expected level of additions to stock as well as from deviations in the expected level of issues from stock. For example, with respect to incoming stock, it is possible that the quantity of a delivery is lower than expected or the delivery arrives late. This is frequently due to deficiencies in the logistic performance capability of the delivery processes but it can also be caused by insufficient procurement planning in the internal company processes. For instance, important planning parameters, such as replenishment times, are often based on estimated values, which are not subject to regular inspection and adjustment. With respect to stock issues, deviations always lead to delivery bottlenecks when an article has already fallen below its reorder level and the demand within the replenishment time is larger than expected. Under such demand conditions, it is no longer possible to ensure delivery capability without a safety stock.

It is of particular importance in industrial practice to consider the question of what target stock levels (BL_{target}) are required in the warehouse in order to attain the desired target service level (SL_{target}). This target service level is generally determined by considering the counter-running inventory and shortfall costs. The mean inventory level that is needed to ensure the desired service level can be determined from the shape of the operating curve. Since the incoming stock size is usually predetermined by upstream processes, the safety stock level proves to be a decisive parameter when it comes to determining a position on the service-level operating curve. The safety stock level required to achieve a certain target service level (SSL_{SL}) is calculated as follows:

\[
SSL_{SL} = BL_{0} \cdot (SL_{target}^{2} - 1) + SSL_{100\%} \cdot \sqrt{1 - (1 - SL_{target}^{2})}
\]

Where: \( BL_{0} = \frac{QRP}{2} \)

Where
- \( SSL_{target} \) = Target safety stock level [units]
- \( SL_{target} \) = Target service level [%]
- \( SSL_{100\%} \) = Safety stock level to reach 100% service level [units]
- \( BL_{0} \) = Lot stock level [units]
- \( C \) = C-norm parameter (\( C \geq 0 \)) [-]
- \( QRP \) = Replenishment quantity [units]

Building on the findings obtained from the service-level operating curve, the following section introduces the dynamic and future-oriented method of calculating the safety stock. The information base comprises discrete demand forecasts, the quality of these, and information concerning supplier reliability. The aim is to employ a dynamic approach to calculate reorder points and safety stocks, these being the central inventory management parameters. For this purpose, all relevant deviations from plan on both the stock additions side and the stock issues side are taken into account.
Deviations from plan on the incoming stock side are reflected by unreliability in external or internal suppliers. This may be in the form of either a delayed delivery or a delivery with insufficient quantities. Since deviations in quantity in industrial practice can be absorbed by short-notice subsequent deliveries or early reordering, these deviations from plan can be neglected in the development of the onward procedure.

**Error! Reference source not found.** illustrates the calculation of the required safety stock on the basis of due-date deviations by presenting inventory levels over time. It is assumed that the demand corresponds with the forecast. The reorder level is reached at the time \( t_0 \) and the order is triggered. If the delivery date \( (t_1) \) is not adhered to, the result is a delayed delivery \( (t_2) \), which means that a safety stock must be maintained. This results from the demand within the deadline (shaded area). In order to cope with all possible delays, the maximum lateness from the previous period \( (L_{max}^+) \) is taken into account. Hence the resulting safety stock is calculated by taking the sum of the forecast from the beginning of the due-date deviation \( (t_1) \) up to the maximum lateness \( (t_2) \) (Becker et al. 2013).

![Figure 3: Safety stock level for due-date deviations](image)

Since extreme deadline deviations cannot be balanced out by means of safety stocks, the maximum positive due-date deviation is, for calculation purposes, taken to be three times the standard deviation of the replenishment time (formula 4). This covers more than 99.7% of possible due-date deviations, and chance extreme values or data errors are not considered. It is assumed that the replenishment time provided corresponds with the statistical mean replenishment time.

\[
SSL_{1\text{min}} = \sum_{t_{f=TRP}}^{t_{f=TRP+3\sigma_{TRP}}} f_t
\]

(4)

Where: \( SSL_{1\text{,min}} = \) Safety stock level for due-date deviations [units]  
\( f_t = \) Daily forecast [units]  
\( TRP = \) Replenishment time [units]  
\( \sigma_{TRP} = \) Standard deviation of replenishment time [SCD]

Besides deviations on the stock additions side, deviations from plan on the stock issues side must also be taken into consideration. Since future demands are estimated on the basis of forecast information, deviations can occur between forecast quantities and actual demand. For this reason, the calculation must also take into account the resultant forecast error. However, the deviation to be considered for maintaining delivery capability only occurs when the actual demand is larger than the forecast demand within the
replenishment time \((t_0 \text{ to } t_1)\) (Becker et al. 2013). In Error! Reference source not found., the required safety stock due to deviations from forecast is shown as the shaded area between the curves of the actual stock issues and the forecast stock issues (dotted line).

**Figure 4: Safety stock level for forecast deviations**

To ensure delivery capability, the forecast demand within the replenishment time must be multiplied by a forecast correction factor (Formula 5). This describes the ratio of the maximum possible forecast deviation to the mean demand. For this purpose, it is first necessary to determine the mean absolute forecast error that represents systematic errors (fundamental over/underestimates). Consideration is then given to the standard deviation of the daily forecast error that describes the dispersion of the deviation. The daily forecast error is defined as the difference between the actual daily demand and the daily forecast. The maximum possible forecast deviation is obtained from the sum of the mean absolute forecast errors and three times the standard deviation of the daily forecast error. Tripling the standard deviation ensures that over 99.7% of all possible incidences of deviation are covered. Moreover, random and unpredictable outlying values are deliberately neglected. With the average demand \(\mu_m\) in the denominator, the forecast deviations are weighted so as to obtain the mean forecast deviation. It is assumed that there is a continuous demand within the period under consideration.

$$SSL_{2,\text{min}} = \frac{\mu_{FC_{\text{est}}} + 3 \cdot \sigma_{FC_{\text{est}}}}{\mu_m} \cdot \sum_{t_0}^{t_0 + TRP} f_t$$

Where: 
- \(SSL_{2,\text{min}}\) = Safety stock level for forecast deviations [units]
- \(\mu_{FC_{\text{est}}}\) = Mean absolute forecast error [units]
- \(\sigma_{FC_{\text{est}}}\) = Standard deviation of forecast error [units]
- \(\mu_m\) = Mean demand [units]
- \(f_t\) = Daily forecast [units]
- \(SCD\) = Shop calendar day

Since due-date deviations and forecast deviations can occur both individually and in combination, the effects referred to are added in marginal cases. Due to the assumption of stochastic independence, the dynamic safety stock \(SSL_{\text{dyn,100%}}\) that takes into account all deviations from plan is obtained by taking the root of the sum of the squares of each of the safety stocks \(SSL_{1,\text{min}}\) and \(SSL_{2,\text{min}}\) (Sachs 2002).

$$SSL_{\text{dyn,100%}} = \sqrt{(SSL_{1,\text{min}})^2 + (SSL_{2,\text{min}})^2}$$
Where: \(SSL_{\text{dyn,100\%}}\) = Safety stock level to reach 100% service level [ME]
\(SSL_{1, \text{min}}\) = Safety stock level for due-date deviations [units]
\(SSL_{2, \text{min}}\) = Safety stock level for forecast deviations [units]

Applying the formulae for the two safety stocks (Formulas 4 and 6) in Formula 6 gives:

\[
SB_{\text{dyn,100\%}} = \left( \frac{t_0 + WBZ + 3 \sigma_{WBZ}}{\mu_{m}} \right)^2 + \left( \frac{\mu_{\text{FC error}} + 3 \cdot \sigma_{\text{FC error}}}{\mu_{m}} \right)^2 \sum_{t=0}^{t_0} f_t
\]  

(7)

Where: \(SSL_{\text{dyn,100\%}}\) = Safety stock level to reach 100% service level [ME]
\(\mu_{\text{FC error}}\) = Mean absolute forecast error [units]
\(\sigma_{\text{FC error}}\) = Standard deviation of forecast error [units]
\(\mu_m\) = Mean demand [units]
\(f\) = Daily forecast [units]
SCD = Shop calendar day

This formula takes into consideration all deviations from plan in a stock and enables a service level of 100%. In industrial practice, a lower service level is generally considered acceptable in order to reduce inventory costs. For this reason, it makes sense to link the formula derived with the findings from the service-level operating curve. Substituting Formula 7 in Formula 3 produces the dynamic, future-oriented method of calculating service-level-oriented safety stocks (\(SSL_{\text{dyn}}\)):

\[
SSL_{\text{dyn}} = BL_0 \cdot (SL_{\text{target}}^2 - 1) + SSL_{\text{dyn,100\%}} \cdot \sqrt{1 - (1 - SL_{\text{target}}^2)}
\]  

Where: \(BL_0 = \frac{QRP}{2}\)  

(8)

Where \(SSL_{\text{dyn}}\) = Dynamic safety stock level to reach target service level [units]
\(SL_{\text{target}}\) = Target service level [%]
\(SSL_{\text{dyn,100\%}}\) = Dynamic safety stock level to reach 100% service level [units]
\(BL_0\) = Lot stock level [units]
\(C\) = C-norm parameter (\(C \geq 0\) [\(\cdot\)])
\(QRP\) = Replenishment quantity [units]

Reorder level is another decisive inventory management parameter that requires temporal dynamic adjustment. It is obtained by taking the sum of the forecast demand within the replenishment time and the dynamic safety stock:

\[
RL_{\text{dyn}} = \sum_{t_0}^{t_0 + TRP} f_t + SB_{\text{dyn}}.
\]  

(9)

Where \(RL_{\text{dyn}}\) = Dynamic reorder level [units]
\(f\) = Daily forecast [units]
\(TRP\) = Replenishment Time [SCD]
\(SSL_{\text{dyn}}\) = Dynamic safety stock level to reach target service level [units]
4. Analysis

The method presented above for dynamically calculating safety stock and reorder levels was derived analytically and should therefore cover all possible deviations from plan encountered in inventory management. Since the dynamic aspect over time is also taken into account, this approach is deemed to have a significant logistic and economic advantage in comparison with other approaches. Therefore, the results are new and contribute to body of knowledge.

To demonstrate the benefits of the new approach and validate the method, it will be necessary to devise and conduct a comprehensive simulation study, in which established methods for calculating safety stocks are compared with the dynamic approach. The evaluation criteria comprise the relevant logistics costs in the form of the mean stock level and the logistical performance in the form of the attained service level. This will permit analysis of the logistical behaviour of an article in a warehouse stage over time under varying framework conditions, conducted in line with a systematic experiment design. Error! Reference source not found. illustrates the experiment design and the schematic flow of the simulation study.

Figure 5: Possible experiment design and schematic flow of simulation study

The (s,q) inventory policy serves as the basis for the warehouse simulation, as it represents the most frequently used inventory policy (Toomey 2002). For the purpose of the simulation, the optimum order quantity q is obtained by applying Harris's EOQ (economic order quantity) approach (Harris 1913).

The implementation of deviations from plan is a central factor when it comes to constructing a simulation environment that highlights the benefits of safety stocks. Furthermore, due-date deviations should be implemented on the stock addition side. Experience made by the IFA in industrial projects has shown that a due-date deviation can be approximated by way of a normal distribution. The systematic choice of mean value and
standard deviation enables the consideration of various types of supplier reliability. On the issues side, there should be deviations in both demand quantities and time intervals between demands. For this purpose, the fluctuations in demand can be represented by means of a normal distribution. To simulate seasonal variations and trend movements, the normal distribution should be superimposed with an oscillation and trend curve. By selecting defined parameters such as the dispersion of the demand rate, amplitude, frequency and trend factor, it is possible to simulate characteristic customer demand behaviours. Moreover, several variations in forecast quality should be implemented. In this way, it will be possible to analyse the relationship between the safety stock level and forecast quality, thus ensuring a validation that is independent of a specific forecast method.

5. Summary and Conclusions

This article presents a method for the dynamic, future-oriented management of stock for single-stage single-item inventory systems with non-stationary demand and replenishment time uncertainty. Based on such data as supplier reliability, forecast information, and the quality thereof, it enables the calculation of an ongoing safety stock curve that is dynamic over time. Moreover, discrete reorder points can be calculated as central parameters of inventory management. To validate the dynamic method and demonstrate the benefits of the new approach, future research activities should focus on devising and implementing a comprehensive simulation study. This should compare established methods of calculating safety stocks with the dynamic approach. The evaluation criteria are logistics costs, in the form of the mean stock level, and logistical performance, in the form of the service level attained. By employing a systematic experiment design, this should make it possible to analyse the logistical behaviour of an article in a warehousing stage over time under various framework conditions.

References

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